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VI Semester B.A./B.Sc. Degree Examination, September - 2021

MATHEMATICS

(CBCS Semester Scheme Freshers & Repeaters 2016-17 and onwards)

Paper : VII

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

Answer **ALL** questions.

PART - A

1. Answer any **FIVE** questions. **(5×2=10)**

   a. If V(F) is a vector space over the field F, and 0 the zero vector of V, then prove that $a.0 = 0 \forall a \in F$.

   b. For what value of k the vectors {(-1,2,1), (3,0,-1),(-5,k,3)} are linearly dependent.

   c. Find the matrix of the linear transformation $T: R^2 \to R^2$ defined by $T(x,y) = (4x+2y, 6x-5y)$ with respect to standard bases.

   d. Define linear transformation of a vector space.

   e. Write scalar factors in spherical polar co-ordinate system.

   f. Solve : $\frac{dx}{y^2} = \frac{dy}{xz} = \frac{dz}{xy}$.

   g. Form a partial differential equation by eliminating constants from $z = a^2x^2 + b^2y^2$.

   h. Solve : $p = e^q$.

PART - B

Answer **TWO** full questions. **(2×10=20)**

2. a. Prove that the intersection of any two subspaces of a vector space V(F) is also a subspace of V(F).

   b. Find the dimension and basis of the subspace spanned by the vectors {(2,4,2),(1,-1,0),(1,2,1),(0,3,1)} of V_3(R).

(OR)

[P.T.O.

3. a. Prove that the subset $W=\{(x_1,x_2,x_3)/x_1^2+x_2^2+x_3^2\le 1\}$ of the vector space $V_3(R)$ is not a subspace of $V_3(R)$.

   b. In an n - dimensional vector space V(F) prove that
      i. any (n+1) vectors of V are linearly dependent
      ii. no set of (n-1) vector can span V.

4. a. Find the linear transformation $T:V_2(R)\rightarrow V_2(R)$ such that T(1,2) = (3,0) and T(2,1)=(1,2).

   b. Find the matrix of the linear transformation $T:V_3(R)\rightarrow V_2(R)$ defined by T(x,y,z)=(x+y,2z-x) relative to the bases $B_1=\{(1,0,-1),(1,1,1),(1,0,0)\}$ and $B_2=\{(0,1),(1,0)\}$.

(OR)

5. a. If $T:U\rightarrow V$ is a linear transformation then prove that
      i. Kernel N(T) is a subspace of U
      ii. T is one - one if and only if $N(T)=\{0\}$.

   b. Find the range space, null space, rank nullity and hence verify rank - nullity theorem for $T:V_3(R)\rightarrow V_2(R)$ defined by T(x,y,z)=(y-x,y-z).

PART - C

Answer any **TWO** full questions. **(2×10=20)**

6. a. Verify the condition for integrability and solve $3x^2dx+3y^2dy-(x^3+y^3+e^{2z})dz=0$.

   b. Solve $p(y-z)+q(z-x)=x-y$.

(OR)

7. a. Show that the cylindrical co - ordinate system is orthogonal curvilinear co-ordinate system.

   b. Express the vector $\vec{f}=yz\hat{i}-2x\hat{j}+y\hat{k}$ in cylindrical co-ordinates and find f_ρ, f_φ, f_z.

8. a. Solve : $\dfrac{dx}{mz-ny}=\dfrac{dy}{nx-lz}=\dfrac{dz}{ly-mx}$.

   b. Solve : $\dfrac{dx}{x^2-y^2-z^2}=\dfrac{dy}{2xy}=\dfrac{dz}{2xz}$.

(OR)

9. a. Express $\vec{f}=z\hat{i}-2x\hat{j}+y\hat{k}$ in spherical polar coordinates and hence find f_r, f_θ, f_φ.

   b. Express $\vec{f}=x\hat{i}+y\hat{j}+z\hat{k}$ in cyclindrical co-ordinates system and hence find f_ρ, f_φ, f_z.

PART - D

Answer any **TWO** full questions. **(2×10=20)**

10. a. Form the partial differential equation by eliminating arbitrary function from $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$.

b. Solve : $p^2 + q^2 = x + y$.

(OR)

11. a. Solve : $[D^2 + 5DD' + 4(D')^2]z = \cos(4x + y)$.

b. Solve : $p^2 = z^2(1 - pq)$.

12. a. Solve by charpit's method $px + qy = pq$.

b. Solve : $\left(D^2 - 2DD' + (D')^2\right)z = e^{x+2y}$.

(OR)

13. a. A tightly stretched string with fixed end points x = 0 and $x = l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement y(x,t).

b. Solve : $\frac{\partial u}{\partial t} = 16\frac{\partial^2 u}{\partial x^2}$ subject to the conditions

i. u(0,t) = 0, u(1,t) = 0 for all t

ii. u(x,0) = x^2-x, $0 \le x \le 1$.